

Suppose that functions f and g and their derivatives have the following values at $x = 2$ and $x = 3$.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
2	8	2	$1/3$	-3
3	3	-4	2π	5

Evaluate the derivatives with respect to x

A) $2f(x)$ at $x=2$

$$\begin{aligned} \frac{d}{dx}(2f(x)) &= 2f'(x) + f(x)(0) \\ &= 2f'(2) \\ &= 2\left(\frac{1}{3}\right) \\ &= \frac{2}{3} \end{aligned}$$

B) $f(x)+g(x)$ at $x=3$

$$\begin{aligned} \frac{d}{dx}(f(x)+g(x)) &= \\ &f'(x)+g'(x) \\ &f'(3)+g'(3) \\ &2\pi + 5 \end{aligned}$$

C) $f(x)g(x)$ at $x=3$

$$\begin{aligned} &f(x) \cdot g'(x) + g(x) \cdot f'(x) \\ &f(3) \cdot g'(3) + g(3) \cdot f'(3) \\ &(3)(5) + (-4)(2\pi) \\ &15 - 8\pi \end{aligned}$$

D) $\frac{f(x)}{g(x)}$ at $x=2$

$$\begin{aligned} &\frac{g(x) \cdot f'(x) - f(x) \cdot g'(x)}{[g(x)]^2} \\ &\frac{(2)\left(\frac{1}{3}\right) - (8)(-3)}{2^2} \end{aligned}$$

$$\begin{aligned} &\frac{2 + 24}{4} = \frac{26}{4} \\ &= \frac{13}{2} \end{aligned}$$

E) $f(g(x))$ at $x=2$

$$\left. \begin{aligned} \frac{d}{dx} (f(g(x))) \\ f'(g(x)) \cdot g'(x) \\ f'(g(2)) \cdot g'(2) \end{aligned} \right\}$$

$$\begin{aligned} f'(2) \cdot (-3) \\ \frac{1}{3} \cdot -3 \\ (-1) \end{aligned}$$

F) $\sqrt{f(x)}$ at $x=2$

$$\begin{aligned} \frac{d}{dx} ([f(x)]^{1/2}) &= \frac{1}{2} [f(x)]^{-1/2} \cdot f'(x) \\ &= \frac{f'(x)}{2\sqrt{f(x)}} \end{aligned}$$

G) $\frac{1}{g^2(x)}$ at $x=3$

$$\frac{d}{dx} ([g(x)]^{-2}) = -2 [g(x)]^{-3} \cdot g'(x) = \frac{-2g'(x)}{[g(x)]^3}$$

$$\frac{d}{dx} [f(x)^2]$$

$$2f(x) \cdot f'(x)$$

F) $\sqrt{f^2(x) + g^2(x)}$ at $x=2$

$$\frac{d}{dx} ([f^2(x) + g^2(x)]^{1/2})$$

$$\frac{1}{2} [f^2(x) + g^2(x)]^{-1/2} \cdot [2f(x) \cdot f'(x) + 2g(x) \cdot g'(x)]$$

$$\frac{[2f(x) \cdot f'(x) + 2g(x) \cdot g'(x)]}{2\sqrt{f^2(x) + g^2(x)}}$$

